STA 610L: Module 2.4

Multi-way ANOVA and interactions

Dr. Olanrewaju Michael Akande

Multi-way ANOVA and interactions

ANOVA can be easily extended to accommodate any number of categorical variables.

Variables may each contribute independently to a response, or they may work together to influence response values.

Interaction effects are important when the association between one independent variable and the response may depend on the level of another independent variable.

Click this link for insight on what interactions imply in terms of group means

Interaction example

For example, suppose that we are interested in a two-way ANOVA model in which the response y is a measure of headache pain, and the independent variables include the type of pill taken j, with $j = 1$ for placebo and $j = 2$ for ibuprofen, and the number of pills taken k, where $k = 1, 2$.

While we may expect lower pain if multiple ibuprofen pills are taken, we would not expect the same decrease in pain if multiple placebo pills were taken.

Consider the model

$$y_{ijk} = \mu + \alpha I(j = 2) + \beta I(k = 2) + \gamma I(j = k = 2) + \varepsilon_{ijk}.$$

Interaction example

$$y_{ijk} = \mu + \alpha I(j = 2) + \beta I(k = 2) + \gamma I(j = k = 2) + \varepsilon_{ijk}$$

In this model, the mean is parameterized as follows.

Drug	# of Pills	Mean
Placebo	1	μ
Ibuprofen	1	$\mu + \alpha$
Placebo	2	$\mu + \beta$
Ibuprofen	2	$\mu + \alpha + \beta + \gamma$

What types of parameter values would we expect to see if there is an interaction in which there is a dose effect for Ibuprofen but not for placebo?

Interaction example

$$y_{ijk} = \mu + \alpha I(j = 2) + \beta I(k = 2) + \gamma I(j = k = 2) + \varepsilon_{ijk}$$

In this model,

- the expected difference in pain level moving from 1 to 2 ibuprofen pills is $\mu + \alpha - \mu - \alpha - \beta - \gamma$
- the expected difference in pain level moving from 1 to 2 placebo pills is $\mu - \mu - \beta$
- the expected drug effect for those taking one pill is $\mu + \alpha - \mu = \alpha$
- the expected drug effect for those taking two pills is $\mu + \alpha + \beta + \gamma - \mu - \beta = \alpha + \gamma$

So no interaction $(\gamma = 0)$ means that the drug effect is the same regardless of the number of pills taken.

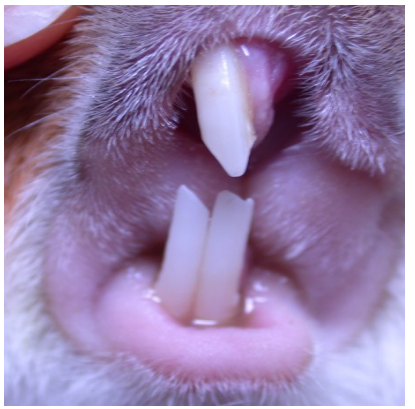
For there to be no drug effect at all, we need $\gamma = 0$ and $\alpha = 0$.

R'S MOST EXCITING DATA

We are going to explore R's most thrilling data -- the famous tooth growth in Guinea pigs data!

Ahh, how cute! Our Dickensian guinea pig has a mystery to solve -- which type of Vitamin C supplement is best for tooth growth!

R's most exciting data



Guinea pig dental problems are NOT fun.

Our dataset (Crampton, 1947) contains, as a response, the length of odontoblasts (cells responsible for tooth growth) in 60 guinea pigs, each of which receives one dose of vitamin C (0.5, 1, or 2 mg/day) via one of two delivery methods (orange juice (OJ) or ascorbic acid (VC)).

Researchers wanted to know if the odontoblast length could be used as a marker of Vitamin C uptake, for the purposes of providing better nutritional supplementation to members of the Canadian armed forces (alas, the first of many injustices for Oliver Twisted Teeth -- the study was not done to help little Guinea piggies).

R's most exciting data

```
library(ggplot2)
gp=ToothGrowth
gp$dose=as.factor(gp$dose)
# Default bar plot
p<- ggplot(gp, aes(x=dose, y=len, fill=supp)) +
  geom_bar(stat="identity", position=position_dodge())
# Finished bar plot
p+labs(title="Odontoblast length by dose", x="Dose (mg)", y = "Length")+
   theme_classic() +
   scale_fill_manual(values=c('#999999','#E69F00'))
```

R'S MOST EXCITING DATA

Looking at the boxplot of the growth data, what type of ANOVA model may be most appropriate?

We will revisit this in the class discussion sesssion.

What's next?

Move on to the readings for the next module!